5

Farida Abdulla Khan

Mathematics and its discontents: How well does Mathematics pedagogy serve the children of the poor?

TRG Poverty & Education I Working Paper Series

Max Weber Stiftung
Foundation
German Humanities
Institutes Abroad

MATHEMATICS AND ITS DISCONTENTS

HOW WELL DOES MATHEMATICS PEDAGOGY SERVE THE CHILDREN OF THE POOR?

Abstract

This paper describes the uniquely specific characteristics of Mathematics, the special significance of semiotic representation systems in mathematical thinking and the special demands it makes on children's cognitive processing for learning Mathematics. I argue that problems of learning in Mathematics classrooms need to be linked to and analysed within the socio-economic contexts of schooling practices and children's lives, so that the burden of failure is not attributed solely to the child and to individual capacity. The status of Mathematics within the educational system gives it tremendous power to determine children's life chances either by way of progression to higher levels of education or by a disproportionately large influence in determining admission to high status courses and professions. Ignoring the special nature of Mathematics, the out-of-school constraints that poverty imposes, and the less than adequate classrooms and schools where the poor have access to education, only serves to mask the complex ways in which learning is mediated by these factors and the serious implications this has for Mathematics learning and life chances of poor children.

Keywords: Social contexts of Mathematics Cognition; Inequities in Mathematics Education; Mathematics and Poverty; Inequalities in Access

Name, contact details including email: Farida Abdulla Khan, Department of Educational Studies, Jamia Millia Islamia, New Delhi 110 025. Phone: +91 11 2410 6414. Email: khan.farida@gmail.com

Bio: Farida Abdulla Khan is Professor in Educational Studies at the Jamia Millia Islamia, New Delhi. Her research interests include the social contexts of Mathematics education; issues of exclusion and marginalisation in education, and political conflict and education.

Introduction

The idea of education as a universally positive good has not come under enough scrutiny although critiques of this accepted view are beginning to be taken more seriously. The fact that education and the certification it provides is the key to securing jobs and other economic and social advantage can hardly be contested. Nevertheless, how educational opportunity unfolds and how and why it fails so many children is not clearly understood. The practice of education and of schooling is embedded within a highly complex social, economic and political context and cause–effect relationships are difficult to trace and to establish. This location and its dynamics influence the outcomes of education in important ways.

An unproblematic acceptance of education as simply and always a means of positive social change masks the ambiguities and the processes that in fact often contribute to the very inequalities and unjust outcomes that it is meant to overcome. A critical tradition that highlights the ways in which this happens, continues to elicit staunch resistance within mainstream educational theory and practice. Althusser (1971) introduced the concept of education as an 'ideological state apparatus' and Bowles and Gintis (1976) made a landmark intervention with their thesis on 'Schooling in capitalist America'. Bourdieu and Passeron (1990) have alerted us to the forms of social and cultural capital that reinforce class hierarchies through the educational system and in fact promote entrenched social inequalities; Shirley Brice Heath (1983) unveiled the middle class culture of schools that silences other voices and disadvantages working class children and Foucault (1991) analysed the ways in which knowledge bestows power and the reach of social and political control that it allows dominant power groups to yield. These theoretical debates on education, schools and society have initiated and allowed more complex readings of education and inequality Ball, 2004).[1]

Issues of access, inequality and exclusion have been a priority in Indian governmental policy and within the educational establishment since Independence and continue to remain a cause for

[1] The new sociology of education in the UK and similar trends in the USA initiated more complex understandings of inequality in education and much current critical research in education acknowledges this tradition. Some prominent figures (amongst many) are Bernstein (1971) and Young and Whitty (1977) in the UK; Apple (1979), Anyon (1980) and Giroux (1983) in the USA. See volume edited by S.J. Ball (2004) for a review.

concern. Social science research has documented the relationship between education and poverty along with other parameters of disadvantage like caste, gender, region and community that influence educational outcomes. Although prior to the 1990s the focus was largely on access and bringing the poor and the marginalised into schools, a more critical engagement with education has been established in the last few decades and a critical sociology of education has begun to address these issues more substantially. Sociologists have contributed to an understanding of institutional contexts and 'have highlighted processes of stratification, differentiation and hierarchies of power that shape the social reality within schools, and influence the construction of diverse "schooled" identities' (Nambissan, 2014: 88–89).[2]

An important indicator of these hierarchies is the differential level of academic achievement where the poor and the socially marginalised are at a disadvantage, and generally the least successful by any conventional measure of academic success. The content and processes of teaching and learning are critical to academic success and constitute a major focus of research for understanding academic achievement. Because education relies heavily upon the disciplines of Educational Psychology and Child Development for explanations of learning, academic achievement along with failure and success are most often attributed to individual effort and ability. Unfortunately, the relationship between sociological concerns and psychological explanations in educational settings has not been sufficiently examined, and linking the cognitive processes of learning in classrooms to the larger sociological contexts of schools and society remains amongst the least researched areas in education. There is however a growing body of research and theory (still on the margins of mainstream education) that explores the social contexts of learning and cognition and the relationship between mind and society. This has been a multi-disciplinary effort that brings together psychology, anthropology, and philosophy, among others, and although not directly concerned with issues of inequality and social justice, it provides a framework and a possibility for addressing them.[3]

Within mainstream education, the social and class dimensions of education are seen as the preserve of sociology and have had little impact on the study of learning, cognition or pedagogy.

[2] See Nambissan (2014) and Nambissan and Rao (2013) for a review of the changing trends of engagement with issues of disadvantage within educational research in India.

[3] See Lerman (2000) for an excellent review of the social turn in Mathematics education.

Few departments of education in India have engaged seriously with important sociological debates in trying to understand issues of learning and academic achievement, and they remain peripheral to the curriculum and research in most university departments of education or the many colleges of teacher education.

Mathematics is an important school discipline for the study of cognition and as a 'powerful knowledge' system, it remains a compulsory and core component of every school syllabus. Why and what kind of Mathematics children need to learn is not a resolved issue, and although its status and importance has not been adequately challenged, there is some amount of questioning and discussion around it.[4] Notwithstanding these controversies, however, school Mathematics has acquired a status that has important repercussions for the academic and subsequently the professional trajectories of students. Its role as a mechanism of social selection is well recognized (Jorgensen et al., 2014) as also of the fact that Mathematics '…plays a key role in the distribution of life chances. For example, there is widespread concern with how Mathematics acts as a "critical filter" in depriving minority and women student's equal opportunities in employment. Philosophical and ethical considerations like these thus have important implications for Mathematics and particularly for educational theory and practice' (Ernest, 1994: 2).

In India, large numbers of children in middle and high school are unable to cope with the Mathematics curriculum and failure in this subject has been a major cause of detention and subsequently for children dropping out of school (NCERT, 2006). Within the extremely unequal and hierarchical system of school education, Mathematics plays an important part in determining chances of retention. It also has the power to deny options of entry into several high-prestige courses in higher education and failure can and does jeopardise life chances for large numbers of children. The patterns are predictable – the more marginalised the population, the less their chances of progress through the educational system.

I propose in this paper to draw attention to what are usually considered purely psychological aspects of learning and knowledge acquisition in the Mathematics classroom and to the ways in which this learning and transaction between teacher, pupil and text is mediated by the socio-

[4] See Khan (2012) for a more detailed discussion of this issue and its implications for the socially and economically disadvantaged child in India.

economic realities of children's lives as also by the material and social contexts of their learning. I subscribe to the argument that schools are meant to provide access to knowledge that is not available in the home or the community, and that Mathematics classifies as what has been described as ‘powerful knowledge’ that requires schools and specialised teachers to initiate and support such learning.[5] Although school Mathematics is a challenge for all children, its implications for the socio-economically disadvantaged child are critical because the failure of learning is most often attributed to the child, the family or the teacher whereas its relationship to structures of poverty and privilege is overlooked. Given its importance in determining school success, I suggest that recognising the complexity of the content of Mathematics and its cognitive processing along with the contexts within which learning takes place, is vital for understanding the added disadvantage it creates for the already disadvantaged.

I begin by describing the distinctive nature of Mathematics, how it differs from other school subjects and what are the features of Mathematics that make it so difficult for school children everywhere. I make a brief reference to the current trends in the psychology of learning and cognition which are able to explain individual differences but have not been successful with explaining group differences. I present some evidence to capture the extremely inadequate condition of the schools that are available to the poor in India and the constraints under which learning is meant to happen. This is done to emphasize how much more difficult the task of learning becomes when it is complicated with the conditions of learning and how pedagogies become ineffective when we fail to recognise the confluence of factors which goes much beyond the student, the teacher and the classroom. It attempts to explain why the difficulties inherent in the subject matter of Mathematics are so much more daunting for children in contexts of poverty and how it creates additional disadvantages for them in classrooms, in schools and in the struggle to advance within the educational system as a whole. I leave open the question of whether this can be overcome by additional resources and more intensive pedagogies, or whether the very notion of school Mathematics and its gate-keeping powers need to be challenged – therefore whether the question is a pedagogic one or a political one.

[5] See Michael Young (2011) on the importance of learning, schools and 'powerful knowledge', especially for the poor.

The Specific Character of Mathematical Knowledge

The special problems that school Mathematics poses for children both at the elementary level and especially at the secondary level, is widely documented. The belief that Mathematics has unique characteristics and that its learning may require different strategies and pedagogies is gaining more acceptance even while it remains contested. This framework allows us to understand why mastery of a knowledge system of this nature is particularly difficult for the average school child. By ignoring this complexity and its implications in the classroom, we mask the intricate ways in which disadvantage is multiplied for children who have no resources other than the school to draw upon.

In making the above argument, I primarily rely on the work of Raymond Duval (1998, 2000, 2006) to elaborate this argument with pertinent examples. Duval's work focuses on unraveling the unique nature of mathematical knowledge and the corresponding cognitive processes involved in its learning and its implications for teaching and pedagogy. The choice, although somewhat arbitrary (given that there is a growing body of research and work within this framework), is made because of its convincing argument for the unique nature of mathematical learning amongst the subjects that children learn in school. To support the argument that learning Mathematics is different and that ignoring this difference masks the very demanding process that its learning entails, I also briefly refer to the work of two other important figures - Paul Ernest, a philosopher and Paul Dowling, a sociologist. Both have engaged extensively with school Mathematics and Mathematics education to support the link between the complexity of Mathematics and its social implications and by extension, especially in the case of Dowling, its capacity to exclude. Ernest (1998, 2003) highlights the fund of unspecified and often ignored 'tacit' knowledge required for learning Mathematics, while Dowling (1998) describes what he terms as the 'high discursive saturation' of the subject matter of Mathematics, its status as a discipline, and the ways in which it creates myths that mask the exclusions that it facilitates. All this has serious implications for Mathematics learning for the poor, who have few resources to meet its demands and little power to challenge its status.[6]

[6] See Khan (2010) for a more detailed discussion of these authors in the context of the social implications of Mathematics education in India.

Difficulties in mathematical comprehension are commonly ascribed to the epistemological complexity of the nature of mathematical concepts but this is equally true of every other domain of knowledge, especially formal knowledge systems that are expected to be mastered in the course of schooling, whether it is the sciences or the social sciences. Duval (2006) ascribes the difference not to the complexity of the concepts alone but to the cognitive activity that is required for mathematical thinking. He identifies the three following characteristics that are unique to Mathematics:

I. *The paramount importance of semiotic representations*: Although all knowledge systems are dependent on semiotic representations, mathematical thought needs semiotic representation as an essential condition. The role of signs in Mathematics is not to stand for some objects, as is the case in other domains, but to *provide the capacity of substituting some signs for others.* In Mathematics, unlike most knowledge domains, there is NO object that can be directly perceived or observed with instruments. Therefore access to mathematical objects is always and *only* through signs or semiotic representations unlike other systems where the access can be both non-semiotic and secondarily semiotic. In Mathematics one can go from one representation to another - the number 'ten' can be represented by strokes or the word 'ten' or the numeral '10'; the mathematical object that is the number itself as a concept is not accessible in any other form and cannot be directly perceived. Thus, for example, the phenomena of astronomy, physics, chemistry, etc. are largely accessible by perception or by instruments (microscopes, telescopes, measurement apparatus, etc.) and in Biology, the semiotic sign or word 'plant' allows us to go from the semiotic representation to the object itself, but a sign in Mathematics never allows this shift to the actual object, *it merely provides the capacity of substituting some signs for others.*

II. *The Cognitive paradox of access to knowledge objects:* This is a specific epistemological situation that radically changes the cognitive use of signs. The crucial question for learners at every stage of the curriculum then becomes: *how can they distinguish the represented object from the semiotic representation used if they cannot get access to the mathematical object apart from the semiotic representations?* The critical threshold for Duval, for progress in learning and problem solving in Mathematics resides in the ability to change from one representation system to another. The ability to recognise that mathematical objects are only accessible through one or another form of representation and to identify them through multiple representations is essential to mathematical thinking. To move from one representational medium to another with ease is the crux of mathematical thinking and should, according to Duval, form the essence of teaching Mathematics in schools.

III. The large variety of semiotic representations used in Mathematics: Mathematics also requires different semiotic representation systems that can be used according to the task and the problem to be solved or the question that is posed. Some processes are easier in one system than another and some can be carried out in one system only. Counting, for example, can be carried out using fingers, or strokes on a paper or, as we do in schools, using the decimal system of numbers and number names. It is fairly obvious that counting of large quantities and especially working with them, is done much more efficiently with the decimal system than the use of a stroke system can allow.

Due to the unique constraints on access to the objects of its study, Mathematics, not surprisingly, is also the domain with the largest range of semiotic representation systems. It uses both systems that are common to other kinds of thinking such as natural language, and those specific to Mathematics, such as algebraic and formal notation. '*This emphasizes the crucial problem of Mathematics comprehension for learners – if for any mathematical object we can use quite different kinds of semiotic representation how can learners recognize the same object through semiotic representations that are produced within different representation systems'?* (Duval, 2006: 108)

Thus we use natural language to refer to the 'three corners of a triangle' but we also represent these as 'x, y, z', and so on. The problem then, according to Duval, arises as much if not more, from these specific ways of thinking as it does from the epistemological difficulties peculiar to the introduction of new concepts. It is essential for students to recognise the represented object through a variety of representational systems that may be very different from each other. For example, the shaded half of a figure (to represent the proportion) in a visual form is a very different kind of representation to the notation '1/2' and the cognitive distance between the two and the effort needed to cover it is barely understood by teachers and curriculum planners.

The case of geometry is a good case in point since it always uses at least two systems of representation. A geometrical figure needs to associate both discursive and visual elements, even though at any point of time only one of these is highlighted. In working with figures like triangles, rectangles and circles, although the visual representation in the form of drawings of the figures is very prominent, especially to the novice and the child, the mathematical properties are represented in statements like 'let ABC be a right angled triangle…'. In a classroom, usually one kind of representation is highlighted and the other left in the background but children are expected to go from one to the other with ease. In actual fact, it is extremely difficult because the common association between shapes and words and the perceptual obviousness goes against its mathematical implications, and intuitive interpretations are often not mathematically appropriate. The essence of mathematical thinking requires comprehension of the ways in which the representation systems function. The difficulty for cognitive processing is not only the representations but also in their transformation. Unlike other areas of scientific knowledge, transformations of signs and semiotic representation are at the heart of mathematical activity.

According to Duval, transformations are at the core of mathematical activity and these are of two types: **treatments** and **conversions**. **Treatments** are transformations that remain within the same register, carrying out a calculation, for example, 2+2=4. The treatment of adding two and two creates its transformation to the number four. **Conversions** are the more problematic of the two and require a change of register without changing the objects being denoted: for example, passing from a natural language statement to a notation using letters. Thus 'two and two is four' uses natural language whereas 2+2=4 uses notation. A child who has not yet learnt to read and write numerals, can count two and two to arrive at four, but cannot be expected to understand the same transformation when confronted with the notational form of the problem.

Multifunctional systems pose enormous challenges for beginner students of Mathematics and for Mathematics educators that need to be identified before they can be addressed.

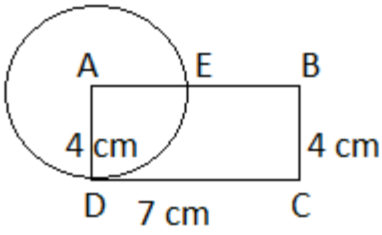
They give the illusion of being more easily accessible whereas this is not the case and leads to a deceptive misreading of a problem. Thus the availability of visual representation for geometric figures and the use of natural language in explaining appear to be an enabling process in early Math learning and has been used as such. Although it is a good entry point for engaging students, it needs extremely deft handling to go beyond one form of representation to another and eventually to think beyond the representations. The problem lies in the mistaken belief that a geometrical figure is the mathematical object itself.

The drawing of a circle on the pages of a book or a note-book, for example, is never the object itself; it is always and only a semiotic representation of a mathematical concept defined as such. But almost always students in primary and elementary Mathematics classes, and very often teachers, make this serious mistake that eventually inhibits mathematical processing and thinking because it also disallows the shift from one semiotic system to another in representing the essential mathematical object: a circle. This flexibility of thinking and transformation in carrying out calculations based on the understanding of the properties of a circle is lost to some extent when the semiotic representation (the visual drawing of a circle) is mistaken for the mathematical object itself. Textbooks, in an effort to make these concepts meaningful, make precisely this mistake and therefore impede mathematical thinking. When children 'see' the visual image of a circle as an object and not as a representation for something else, progressing to solving mathematical problems and thinking of circularity and its properties mathematically is likely to pose serious problems.

An example that illustrates the complexity of this phenomenon is cited by Duval in a study carried out with children in middle and primary schools in France (Duval, 2006: 118). A geometrical problem was presented to children entering middle school which, according to their curriculum, they are expected to solve at this level. The problem was presented to thousands of school children but even at the middle school level very few were able to solve it correctly (and yet French children's scores are among the highest in comparative studies of mathematical achievement in international comparisons). The children's performance illustrates how 'seeing' in geometry can actually take away from the mathematical discourse required and how this dissociation can actually hinder mathematical comprehension rather than facilitate it.

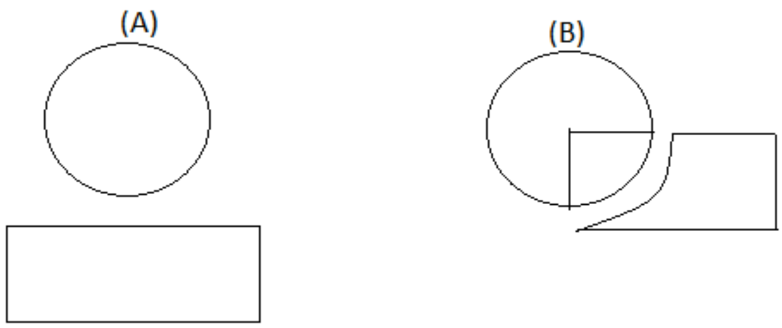
Presented with a diagram (Figure 1), children were asked the following question: In the figure sketched here representing a rectangle ABCD and the circle with centre A, find the length of segment EB.

Figure I Geometrical problem presented to children



Children gave a number of answers but only about 9–20 per cent (of roughly 5,000 children) were able to give the correct answer. Figure II represents two ways of identifying the two sub-figures within the original figure – representation A and representation B (Duval, 2006: pp. 117, 118).

Figure II Two sub-figures that represent the two different ways of viewing the problem



To find the mathematical answer for this problem, however, students needed to look at the two sub-figures in B and not the two subfigures in A. The sides of the rectangle here for mathematical purposes need to be seen as the radius of the circle and NOT as sides of a rectangle. Along with knowledge of the properties of circles and rectangles, the child is required to concentrate on the one-dimensional aspect of the rectangle and its sides and although this sounds simple, it needs considerable and conscious effort.

Our school experience and exposure to math texts intuitively encourages the first kind of identification – a circle and a square. Circles and squares in our classrooms are usually presented as three dimensional and often in the shape of objects. Although this is a good entry into understanding shapes, the representational character of the objects needs to be abstracted and conceptual clarity established. The multiplicity of transformations that is needed for what looks like a fairly simple problem is the essence of mathematical thinking. Teachers and schools need to facilitate this if children are to become mathematically competent and to think mathematically. Acquiring knowledge of the systems of representations is an important first step but it is equally important to learn to manipulate and to use them effectively. Research in Indian classrooms provides evidence of the fact that children in middle school, although able to solve routine textbook problems in geometry, have fuzzy understanding of the transformations and very little clarity about the solutions (Kaur, 2010). These special features of Mathematics set it apart from other school disciplines and therefore call for serious rethinking of how the already ambitious and accelerated school Mathematics curriculum is to be negotiated.

Mastery of the semiotic representational system becomes an important tool for carrying out operations and knowing how to work with the tools that provide solutions to a mathematical problem. The tools for carrying out transformations are a critical component of mathematical thinking that come from a cultural corpus of a long intellectual history and cannot be instantly mastered or spontaneously 'constructed'. The variety of representation systems and the specific capacity for each to perform certain mathematical processes poses very specific problems and needs specific support, sustained exposure and specialised training.

The work of Paul Ernest (1998, 2003) is useful for understanding this analysis because it elaborates the relations between personal and public knowledge and places Mathematics classrooms within an epistemological perspective. He states that Mathematics, like other disciplines, has its own very specific disciplinary discourse that must be mastered for initiation into the 'community' of professional mathematicians and competent Mathematics students. Knowledge in Mathematics is never wholly explicit; although it is often characterised as a collection of validated propositions, which in the case of mathematical knowledge means mainly a set of theorems with proofs. 'Most personal knowledge,' he states, 'is not of the explicit propositional sort, but consists of tacit knowledge of methods, approaches and procedure, which can be applied in new situations and to problems' (Ernest, 1998: 249). The tacit nature of this knowledge becomes more problematic because access to the knowledge objects is by its very nature, never direct.

He claims that standards for proof and definition in Mathematics can never be made fully explicit; rather, this is done in the form of exemplary problems, solutions and proofs. Often these will be exemplified in texts rather than in explicit statements. Mathematics therefore includes an important body of unspecified knowledge which is acquired largely through immersion in a mathematical community. 'Not only does it mean that mathematical knowledge extends beyond the explicit to include a tacit dimension but that beyond the abstract and general knowledge of the results, methods, and language of Mathematics there is an important concrete knowledge of Mathematics but which is also very specific and specialised. This includes knowledge of instances and exemplars of problems, situations, calculations, arguments and proofs, applications and so on. Knowledge of particularities as well as generalities thus plays an important part in Mathematics' (Ibid.: 250) This may also explain why total immersion in seemingly mindless activities of solving hundreds of given Mathematics problems (a common practice for all serious students within the Indian education system, especially those preparing for entrance exams for institutes of technology) facilitates this mastery in some cases. A long period of apprenticeship and deep immersion in the language and practice of mathematical processes is an essential requirement for learning to think mathematically and novices need to be carefully guided and supported at every level. For children of professional and educated parents and in most middle class homes, academic support comes in a variety of ways – help with homework, additional books, games, puzzles and now of course computers, and academic support programmes that give children the immersion into this language that is not possible within the classroom. For those without access to all these supplementary systems, it is an uphill struggle.

Paul Dowling (1998) makes the point that the discipline of Mathematics is a highly specific and well-guarded field of knowledge where entry is limited and often puzzlingly difficult. He uses Bernstein's notion of 'restricted coding orientation', the propensity to generate meanings that are highly context dependent and 'elaborated coding orientation' that refers to an inclination to context independence. This clearly resonates with the situational/abstract thinking divide and both dichotomies, the concrete/abstract or the local/general, have a modality of the discursive in that they depend on language. The difference however is that situational thinking is associated with low discursive saturation and abstract thinking and elaborated coding with high discursive saturation corresponding to context dependency and context independence. Mathematics is clearly a case of high discursive saturation, an activity which is highly organised at the level of discourse and therefore produces generalized utterances. Domestic and manual activities on the other hand are examples of low discursive saturation and therefore produce localised utterances.

He goes on to state that practices exhibiting high discursive saturation are at highly complex levels of discourse and exhibit comparatively complete articulation. They are also highly organised and this makes it possible for them to produce generalisations. Utterances of the low discursive saturation type are characterised by implicit regulating principles, are context specific and must be interpreted within the context of a particular activity (Dowling, 2003). He describes Mathematics as an intellectual activity that is both exclusive and elusive, and which maintains an elite character in the confines of that very elite institution – the University. He refers to Mathematics as 'a mythologizing activity to a degree that is perhaps unparalleled on the school curriculum' (Dowling, 1998: 2) and condemns the ascription of what is essentially non-mathematical activity to be designated as mathematical, which is exceedingly misleading for students and only serves to create illusions of learning. Drawing a visual pattern, or weaving a basket, he maintains, may contain elements of calculation and mathematical thinking but they do NOT qualify as Mathematics, nor will they in and of themselves initiate children into the formal disciplinary knowledge of Mathematics. Naming these everyday activities as Mathematics only takes away from its abstract and complex nature and unnecessarily lulls children into a false sense of achievement.

The use of mathematical assessment to evaluate children's academic status while ignoring the cognitive complexity of mathematical processing, or the severely limited nature of the institutions within which it is transacted in conjunction with the material conditions of children's out-of-school experiences, only serves to hide the unequal nature of schooling and its consequences.

Mathematics Education and the Psychology of Learning

Notwithstanding the complex nature of Mathematics learning and the problems that Mathematics educators struggle with worldwide, difficulties of learning are seldom analysed within the broader perspective of socio-economic contexts and solutions are sought within a narrow cognitive understanding. Presented below is a brief account of the general trends within Psychology that have influenced Mathematics education in India to appreciate its limitations. Although a more serious discussion of these trends would be useful, it is beyond the scope of this paper.

Psychologists by and large concern themselves with the child, the teacher and the classroom to explain differential levels of learning, and academic achievement is attributed to individual differences. That these variations also tend consistently and systematically to vary along lines of social and economic disparity has been largely ignored. Issues of inequality, discrimination and exclusion and their sociological aspects, although recognised, are seldom taken into consideration in explaining learning and knowledge acquisition.

Education has relied on a long tradition of behaviorist explanations of learning that continues to hold sway in classrooms and pedagogy despite the progressive turn to a more child-centred approach. Piaget's genetic epistemology and notions of the child's 'construction' of reality (Piaget, 1958; 1965) caused an important shift in the psychology of learning and ideas about children's thinking that seriously influenced the field of education. As the theory became more widely known, the educational imagination was most captured by the idea of the child as 'active' learner and the teacher as 'facilitator'. Piaget's primary focus of exploration was the genesis of thinking, logical reasoning and human intelligence. The mind of the child was the medium through which this trajectory of 'genetic epistemology' and the shift from childhood to adulthood could be best observed. That the child is also deeply embedded in a social, economic and political world as well as other aspects of development was not critical to his immediate intellectual project.

Largely as a result of this theory but also of other progressive strands in education, efforts at providing children with experience, practice, and freedom to explore and to construct knowledge have gained currency in educational practice. A deeper and nuanced understanding of how this facilitates knowledge acquisition has not been so easily understood. In large numbers of classrooms the world over, this shift has not been able to yield results in terms of improving the learning of vast bodies of knowledge that children are expected to master when they attend school. These problems persist despite efforts to initiate curricular reform and child-friendly pedagogies based on this theoretical framework, as was attempted with the NCF 2005 in India.[7]

Within Psychology and the cognitive sciences, mathematical thinking, as a close to perfect example of rational thinking, has been a popular area of research and provides rich data for testing and verifying assumptions about human cognition. Because of its rational and abstract nature, it is also seen as least influenced by social or cultural factors and its learning is sought to be explained in purely cognitive terms. All children in school are expected to learn Mathematics and yet the difficulties they experience in doing so remains a challenge for teachers and researchers and is a major reason of children's alienation from learning and classroom activity. It also, more than most subjects, marks children as academically competent or not. Rapid advances in technology have enhanced the prestige and status of the discipline to the extent that nations worldwide are becoming aware of the considerable challenge they face in the quest to improve Mathematics teaching for their students (English, 2002)

The evidence of a strong relationship between Mathematics achievement and socio-economic factors has inspired a corpus of research which, according to Valero and Meaney, (2014: 977), has a 'taken for granted' character that is accepted uncritically. It is only in recent years that scholars in the field of Mathematics education are beginning to unravel the complex relationship between Mathematics classrooms, the cognition of Mathematics learning, socio-economic factors and poverty (Zevenbergen, 2000; Lerman, 2000).[8] At the global level, there is support for the hypothesis that socio-economic factors have a significant impact on Math achievement whether it is at the level of students, the schools or the country (Jurdak, 2014).

Research on Mathematics education has expanded vastly and produced a rich body of work but the persistent disadvantage in academic achievement and its relationship to social and economic

[7] Banerji's work, in press, supports the argument and Minocha (2013) provides some interesting data.

disadvantage is yet to be adequately explained. This is an issue of concern amongst a small but growing community that is veering Mathematics education to focus more closely on unraveling 'more nuanced approaches for understanding the social, political and historical constitutions of these relationships' (Valero and Meaney, 2014: 977).[9] This paper, as part of this effort, is a call to look more closely at the relationship between the cognitive processes of mathematical learning and the constraints that limited resources impose.. Although the relationship plays out more closely in schools and classrooms, understanding it in a broader context brings the burden that poverty imposes on children out into the open, and places the responsibility for resolving it on the educational structure, the state, educational policy and educational spending.

State of Schools and the Contexts of Teaching and Learning:

Given that theories of learning and cognition have not been able to resolve the persistent problems of under achievement observed amongst the poor and the marginalised, I suggest that exploring and understanding the ways in which Mathematics learning is mediated by its subject matter and the material conditions in which it takes place, are likely to provide some answers. Introducing the material conditions of schools and classrooms forces us to confront child poverty and the deprivations that impact the lives of the poor. Leaving those out of the equation of learning and academic achievement only masks the inequalities of provision that are so deeply implicated in the academic outputs, and on which the privileges of successful schooling are contingent.

Concerns around school education in India became prominent in the 1980s when efforts for universalisation of education started to gain momentum. Increased access and the Right to Education Act (RTE), 2009 have forced issues of quality to be taken more seriously, especially in the government school sector that is now more or less the preserve of the poor and the deprived. The inadequacies of the system—whether it is in terms of availability, infrastructure, teacher education, teacher indifference, pedagogy or curriculum and textbooks—have been widely reported and discussed although a closer engagement with classroom practice has traditionally

[9] The *ZDM Mathematics Education* (2014) issue 46, published online, is an excellent collection of articles that represent a vibrant debate around issues of Mathematics, education, schools and society.

remained within the domain of Psychology. It is in the context of this systemic failure that the 'failure' of the underprivileged child needs to be better analysed and understood.

The implications of what and how children learn is of central concern to the project of schooling and a long tradition of research and theory has contributed enormously to its understanding. Although this concern is critical for every child in school, its importance is paramount for children who enter schools with great expectations and aspirations of changing their lives. The purpose of this section is to highlight the importance of the contexts within which disciplinary knowledge is acquired and to acknowledge the extremely impoverished environments that are available to the poor and the marginalised. It is not my intention to dismiss government schools or the many efforts of teachers, students, local officials and communities that struggle to educate more and more children in increasingly difficult circumstances. On the contrary, given that the system still provides education to more than half the child population in this country, its importance and impact cannot be overlooked. Its problems and shortcomings need highlighting precisely because it has serious implications for the education of millions of children – mostly powerless and disenfranchised. Unravelling the complex ways in which this provision of schooling mediates the subject matter of Mathematics in conditions that fail to support serious learning, allows at least a glimpse of the burden that the poor child carries in trying to compete and to succeed at what she is falsely led to believe is possible within the promise of schooling.

The hierarchical and segmented nature of the Indian school system is well documented. A two-tiered system of schooling that divides a variety of private schools catering to the elites and Government schools for the middle and lower classes is slowly being eroded not only by a slew of private schools of varying grades but also by a grading of schools within the Government school system itself.[10] There is a steady curtailing of investment in the schooling of the poor and indirect subsidising of private schooling of the middle classes that deprives the poor of schooling and contributes to the reproduction of social inequalities (Abraham, 2006) and Government schools are now left with poor girls and very poor children (Manjrekar, 2003). 'The school(s) attended by the children of the rich in many developing countries provide computerised learning environments not dissimilar to those found in best British schools. But these are not typical.

[10] See the edited volume by Chopra and Jeffery (2005) for a number of interesting case studies that exemplify the differentiation within the system of Indian school education.

More typical are those which are struggling to provide a minimum quality of learning against a background of diminishing resources, a rural economy and often still a pre-literate population' (Little, 1998, as quoted in Majumdar and Mooij, 2012).

Much of this is critically determined by public funding and the lowest rung of Government schools, now subscribed to only by the poorest children in the country, remain poorly provided. School education in India has been described as the 'weakest brick in the pillar, with unequal access, poor infrastructure, high pupil teacher ratio, teacher absenteeism and high drop-out rate' (Bedamatta, 2014). The failure of the Indian state to provide universal access to quality schooling and to ensure access to all socio-economic groups has been viewed as '... surely the more dismal and significant failures of the development project in the country' (Ghosh, 2011). Public spending on education in India, despite the rhetoric surrounding it, remains below 5 per cent of GDP, a lower ratio than many developing countries.

Access to school education in India has increased dramatically in the last few decades and the Net Enrolment Rate for children of the 6–14 years age group increased to 99.89 per cent in 2010–11 from 84.53 per cent in 2005–06. However, drop-out rates remain a concern and retention levels for the poor are low. Educational disadvantage includes children who have never enrolled in school but also those who enroll but never complete even the primary cycle as well as those who complete a few or even several years of schooling but are unable to benefit from the schooling process. Poverty is an overarching factor that disadvantages children in schools and a sharply etched divide in schooling is a stark reflection of wealth and privilege along a number of axes such as gender, caste, region and religious community among others (Bandyopadhyay and Govinda, 2013).

Despite several large-scale governmental efforts, there is increasing evidence of low standards of academic achievement and learning all over the country. Performance levels for basic academic abilities that schools are intended to impart – reading, writing and arithmetic – are abysmal. According to the ASER data, 2011, '… while the student enrolment in rural India has seen a rise (96.7 per cent) in the year 2011, a mere 53 per cent of grade 5 children in rural India could read a grade 2 level text and 36 per cent could solve a three digit by one digit division problem' (ASER, 2011). In 2014, the findings show that half the children in

Standard V are barely at proficiency levels of Standard II; of all the children enrolled in Standard V, about half cannot read even at the Standard. II level and about half the children have not learned mathematical skills that should have been acquired in Standard II. Not surprisingly, the worst indicators pertain to children from the most disadvantaged groups both socially and economically.

The following description of a Government Junior High School in Manipur in 2014 is not an exceptional representation of the state of lower-end government schools across the country:

While a small portion of the school building is made of bricks, the rest is made of wood and mud and parts of it are broken. The roof is made of tin, constructed years ago, and the leaks in the roof result in the destruction of walls, furniture and the limited teaching learning aids in the classroom. Classrooms are dark, dingy, dusty and littered with wastepaper. Ventilation is extremely poor with insufficient windows that are partly broken and require urgent repairs. The veranda gathers moss and is cracked.

"Classroom walls are cracked and have cobwebs. There are sufficient classrooms but are partitioned by torn curtains. The available benches, tables and chairs were in good condition in some classrooms but damaged and insufficient in others. The table and chairs meant for the teachers were in disrepair….
… Sadly, the blackboards were no longer usable as there were white patches and what was written on them could hardly be seen. The school lacks even modest aids such as globes, charts, Mathematics and science kits, a library, children's book bank, game materials, and even school textbooks and stationary were inadequate. As is common in schools all over India, there are not sufficient toilets and in the absence of safe drinking water, a few children from the school are sent to carry about forty litres of water to a distance of about a kilometer in the village" (Salam, 2014).

Variations on the above are an accepted feature of schools in urban slums, in remote areas, and all manner of schools catering to children from marginalised economic and social backgrounds. Thus, for example, in my own observations, a government primary school for girls situated in a fairly well-to-do middle-class area in the capital and attended by working-class children of that area, shared several of the features mentioned above (lack of furniture, broken windows, lack of water) and also had its electric supply cut off for almost a week in the peak of summer with temperatures soaring above 35 degrees centigrade, due to a complicated system of payments that the government has put in place. This, I was informed, is routine practice and, in addition to the extremely uncomfortable physical conditions it creates, imposes disruptions in teaching and other school activities. Children are moved to the lower floors (for cooler rooms), several classes are crammed into one room, with little place for children to sit, to spread their books or to have a quiet or comfortable working environment.

The increased access to elementary education and the mass entry of the poor and marginalised into the government school system has also raised questions about teacher-pupil ratios, adequate teacher training and teacher attitudes towards teaching and students. As the most 'expensive' part of the education system, the role and status of teachers remains fluid and limited funding often strikes at the expenditure associated with them. Several alternatives have been put in place to cut costs and large scale recruitment of para-teachers (under different names) around the country has brought in and legitimised a cadre that are expected to have minimum academic qualifications and no pre-service teacher education (Sarangapani, 2011). Although this is a fraught issue in the political economy of education and too complex to be discussed here, it nevertheless undermines and trivialises the practice of teaching and the professional qualities and commitment required for school teaching. Expansion in the recruitment of teachers lags far behind the expanding enrolments of children and the percentage of single teacher schools continues to be high in several states. Research also shows that the percentage of single teacher schools is higher in blocks populated by the most marginalised communities (Rana, 2006)

Given that first generation learners also require most support at the primary and elementary school levels, it is interesting to note that this sector of teacher education has been the most neglected in India. Centralised and rigidly structured curricula, close monitoring and increased assessments, rather than investment in teacher education, especially in the neo-liberal era, are seen as the panacea for improving the quality of schooling. Focusing on 'competencies' and 'subskills' instead of knowledge and reflection reinforces the mechanical role of the teacher. In addition to the poor physical infrastructure of the schools, teaching competence of the quality required, is not seen as a priority (Sarangapani, 2011; Batra, 2013). This has serious implications for Mathematics learning where clarity and understanding of basic concepts is critical for more advanced learning.

Teacher education in India has been criticised for its superficial and largely outdated nature, as well as its duration, curriculum and structure. Teacher education programmes have evoked serious concern whether it is at the level of disciplinary rigour, the inability to engage with the complexities of the social composition of the Indian classroom, or to engage with subject understanding of teachers and students, among other issues. Pupil-teacher ratios (due to a variety of factors) are a major problem and teacher attitudes towards the poor and the under-privileged are highly problematic. The higher class and caste backgrounds of the teachers influences their perceptions and often an ignorance of the life conditions of the students reinforces stereotypes about the lack of support that children receive within their homes, apathy of parents, and so on (Talib, 1992: 87). School teachers, although much maligned, are also trapped within structures where they find themselves marginalised, with little room for autonomy whether it is in classrooms, schools or the larger educational system (Batra, 2006).[11] Teachers function as petty bureaucrats and in the already understaffed government schools, their services are used for a variety of administrative and other tasks so that little time is available for teaching and academic activities. Teachers have low social standing, they are not considered an intellectual cadre and there is little faith in their capacity to think, to take academic decisions or to engage with matters of policy (Kumar, 2008).

The diversity of children in the government school system, the expectations of teaching to a curriculum, within a time frame, the standardisation of systems of testing and assessment all contribute to classrooms that have little scope for creativity, for exploring multiple solutions to problems, working with children individually or for meaningful interactions of a kind needed, especially for children for whom the classroom materials are unfamiliar. The problems of an 'overambitious curriculum' (Pritchett and Beatty, 2012) and the 'Marks Race' (Majumdar and Mooij, 2012), makes additional demands on learning, leaving less room for meaningful teaching or thoughtful and calibrated pedagogies that are consistent with the difficulty levels of the content of learning. Majumdar and Mooij (2012) describe the achievement model used in schools that 'interprets knowledge, comprehension and conceptual understanding in terms of test

[11] MHRD carried out Joint Review Mission of Teacher Education in several states across India in 2013–14 and the reports are available on the following website: http:/www.teindia.nic.in/rjm.aspx. These reports are a rich source of data regarding the many problems of the government school system and of teacher education across India.

scores' (p. 214) and comment upon 'the low quality of education that children receive in many rural primary schools in India, on the heavy reliance on memorization and cramming in these schools, on excessive examination orientation and the impact this has on guiding the entire process of teaching and learning, curricula, teacher involvement, parental aspirations and student activity or passivity' (p. 216).

Schools are more focused on completing the curriculum rather than delivering learning, the onus of which falls on the parents and the home. According to Majumdar and Mooij, 'The privileged have a shortcut to material success due to a lot of cultural capital at their disposal and middle class children squeeze through the grinding system because of massive home support' (p.233) but 'Many from the underprivileged sections of society, unaided by all kinds of middle class home support, are unable to put up with the burden of non-comprehension and hence eventually drop out of the system. … they therefore suffer more due to the undue standardisation. Described as 'failures', they are pushed out of the wasteful schooling system, without gaining anything much and possibly losing their traditional skills' (p. 234).

Given these constraints, classroom teaching is rendered uninspiring and monotonous. Teaching to a syllabus and a standardised examination system leaves little room for innovation or creativity and promotes conformity and discipline. Classroom processes consist of mechanical routine exercises such as teachers reading out from textbooks, giving dictation to students, children being asked to copy passages or math problems from books, chorus repetitions and recitations, etc., the most common activity in the classroom is rote oriented learning and question answer sessions where answers are well defined with little room for manoeuvre' (Majumdar and Mooij, 2012). Mathematics classrooms have been described as 'defined by tyranny of procedure and memorization of formulas' where, 'given the criticality of examination performance in school life, concept learning is replaced by procedural memory'. Partial knowledge is not seen to be acceptable and there is little room for play in answering questions (NCERT, 2006).

Mathematics, more than most disciplines, relies heavily on the teachers' understanding of Mathematics, clarity of concepts and pedagogic techniques. The lack of interaction and communication between teachers at different levels of schooling, and also with Mathematics researchers and university teachers, leaves elementary teachers ill-prepared for facing the problems that challenge them in Mathematics classrooms. Moreover, textbook-centred pedagogy limits initiative and curbs experimentation. Mathematics teachers make extensive use of the chalk and talk method where problems are presented, children are asked to work them out and the solutions are worked out on the blackboard. Teaching aids are rarely available or used and mechanical drill and practice methods fill the limited time the classroom provides (Rampal and Mahajan, 2003; Khan, 2004). The lack of conceptual knowledge of Mathematics amongst students and often even the teachers has been documented in schools throughout India. These methods of teaching are least conducive to student populations who are less than regular in attending school and have little support outside schools to fill in the gaps.

The argument for the especially complex nature of school Mathematics learning and its consequences is meant to demonstrate how schooling is set up to 'fail' certain children and to challenge schools and society on how the playing field can be made level. Although confined to the classroom, this problems of 'failure' and 'success' needs to be addressed not only within the classroom and the school, but at a variety of levels—within the structures of State, education and society—to answer how it is allowed to determine access to knowledge, to higher education, to opportunity and ultimately to life chances. This calls for an engagement from Sociology and the social sciences, but as Nambissan remarks, 'the Indian school and classroom is the most under-researched area in the Sociology of Education in India' (2013: 83). She also notes that '… that the neglect of the study of schools has led to a glossing over of complex processes that mediate school experiences and influence learning of children. This is particularly important in relation to the education of children belonging to the most marginalized groups in Indian society' (Ibid.)

The relationship between school funding, child poverty and achievement in Mathematics was meticulously analysed and perceptively discussed by Payne and Biddle in 1999 when they reviewed large data sets and earlier research in the area. Although their study focused on the USA, their findings and observations are equally relevant for India and other countries where funding of school education is differentiated and social support systems are fragile. The authors describe how, 'The homes of poor children provide little access to the books, writing materials, computers, and other supports for education that are normally present in middle class or affluent homes in America. Poor students are also distracted by chronic pain and disease; tend to live in communities that are afflicted by physical decay, serious crime gangs, and drug problems, and must face problems in their personal lives …. What this means is that poor children have a much harder time in school than either affluent or middle class children' and confirm that 'poor children are also likely to attend badly funded schools, so the raw statistics indication that their achievements are low may also reflect the inadequacies of those schools' (Ibid: 7). The authors conclude that 'of all aspects of home (dis)advantage that one might study for impact on achievement, child poverty is surely a leading candidate' (Payne and Biddle, 1999: 7)

Highlighting the highly strained conditions of schools and formal education that the state provides and the poor avail of, should help us become aware of inequalities and unequal systems that allow children of the privileged classes to 'succeed' in school (however that success may be defined) while countless others (largely the poor, but also disadvantaged on a variety of indicators like caste, gender, community and so on) are absolutely overwhelmed by it. Pedagogies therefore cannot be confined to the subject matter of a discipline and to the proclivities of a 'child'; we need to recognise that learning and academic achievement is deeply implicated in the material conditions of schooling and the structural constraints of the system and constantly mediated by it.

Conclusions and Implications

Learning Mathematics has long been a source of anguish for children, for teachers and for parents, especially after the rudimentary elements of counting, and the simple operations of addition and subtraction are mastered. It is especially difficult for children who have limited means, apart from the school and the classroom, to access this extremely abstract and formal system that forms an important barrier to be crossed in order to advance within the school system. Children may dislike some subjects or find others uninteresting (History as a school subject seems to be a case in point), but given a basic competence of reading and comprehension, a little extra effort (even if it means some rote learning) enables them to 'pass' these subjects. The description of mathematical thinking presented here is meant to underline its exceptional nature and the fact that if the foundations of the discipline, its language and its conventions are not mastered systematically, it becomes increasingly more complex and finally incomprehensible.

Although the benefits of schooling and a school education cannot solely be measured in terms of academic achievement and proficiency scores, they have become important indicators of success and powerful assets for access to further education, jobs, etc., and extremely important for large numbers of the less privileged and the poor who send children to school at great cost in the hope of securing better lives and livelihoods. The role of those with the power to determine what counts as important knowledge has led to suspicions about the role of knowledge in school curricula and as a result' 'the question of knowledge and the role of schools in its acquisition has been neglected by both policy makers and by educational researchers' (Young, p. 10). I have tried through this analysis of the learning of Mathematics to emphasize that it is important for children to acquire knowledge and to understand the seriousness that such learning requires. The filtering capacity of Mathematics is well known and if it is going to determine life chances then it is the responsibility of schools to provide academic, infrastructural and emotional support to learn important Mathematics. It is not my brief to make every child into a mathematician but it is only fair to accept that if the promise of education is made to all children then some promise of equal delivery must be assured and resources made available in an equitable manner.

Disciplines taught in schools have long cultural histories and accumulated knowledge bases that children are expected to master in the course of schooling. Learning and teaching of important and valued knowledge needs sustained effort, time and resources and this places the underprivileged and the poor at serious disadvantage. The examples from Mathematics presented earlier serve to make us aware of the fact that knowledge which counts as Mathematics is valued as a discipline and is likely to determine entry into higher education and into several high prestige occupations, is not easily acquired. It requires immersion, time, effort and imaginative teaching techniques that very few schools are able to presently provide. This is true of all disciplines to a lesser or greater degree and constitutes a handicap that a majority of schools are not able to address, except a few elite and immensely resource rich ones. To overcome the handicap, the affluent and middle classes use various forms of advantage—whether it is material resources or the social power that they wield—neither of which are available to the poor.

The descriptions of schools that have been presented in this paper illustrate the extent of deprivation that poor children face, notwithstanding several grand schemes and policies launched by the Government of India. Lack of funds, teachers, material resources and very often of even classrooms or essentials like a blackboard, make serious and engaged teaching and learning a difficult challenge. Middle and upper class children are able to overcome these hurdles firstly by opting for better equipped and better managed private schools, but more importantly and less visibly through out-of-school support systems. These children have access to educated parents, tuitions, books, games, educational toys and, increasingly, to technological aids to learning. They are also much more likely to have space and time (probably with supervision) to study after school, privileges that poor children lack. Research evidence suggests that children's performance, whether in government or private schools, improves if they take tuitions and confirms the positive role of supplemental help in raising levels of performance on basic academic achievement in terms of reading writing and arithmetic scores (Banerji and Wadhwa, 2013).

I also argue here that psychological theories and the notion of the child's construction of knowledge as interpreted in educational discourse have compounded this discrimination. The turn to child-centred learning although well intentioned, actually draws attention away from the importance of the systems of knowledge and their long histories of accumulated wisdom as well as the importance of a serious pedagogical commitment that is needed to master them. The normative subject of schooling conforms to the average middle class child with a rich fund of cultural capital that supports and complements the project of schooling and enables her to access the system with far less tension than the working class child for whom life and school do not seamlessly complement each other. We need to recognise the enormous burden that schooling imposes on working class children and the inadequacy of what is available for them to achieve success. Unless we search for possibilities of overcoming it, whether through increased resources, better teaching and pedagogy or through the very reconstruction of education to allow multiple routes of learning towards more fulfilling and rewarding lives, the differential benefits of education in an unequal social system are only likely to be exacerbated.

References

Abraham, L. (2006). 'Access to and Outcomes of Secondary Education for the Urban Poor: Findings from a Study in Mumbai City', *Rajagiri Journal of Social Development*, 2(2): 192–213.

Althusser, L. (1971). 'Ideology and Ideological State Apparatuses', in L. Althusser (ed.), *Lenin and Philosophy and other Essays*. New York: Monthly Review Press, pp. 127–86.

Annual Status of Education Report (ASER) (2005–13). *Annual Status of Education Report.* Delhi/Mumbai: ASER Centre/Pratham. http://www.asercentre.org (Accessed 24 May 2014).

Anyon, J. (1980). 'Social Class and the hidden curriculum'. http://www-scf.usc.edu/~clarkjen/Jean%20Anyon.htm, accessed on 24 May 2015

Apple, M. W. (1979). *Ideology and Curriculum*, London: Routledge and Kegan Paul.

Ball, S.J. (2004). 'The Sociology of Education: A Disputational Account', in S.J. Ball (ed.) *The Routledge Falmer Reader in Sociology of Education.* London and New York: Routledge. pp. 10–12.

Bandyopadhyay, M. and R. Govinda (2012). 'Achieving Universal Primary Education: Expanding Access with Equity', in C. Sleeter, S. B. Upadhyay, A.K. Mishra and S. Kumar (eds.) *School Education, Pluralism and Marginality: Comparative Perspectives.* New Delhi: Orient BlackSwan.

Banerjee, R. (In press). 'Students' Understanding of Algebra and Curriculum Reform', in B. Sriraman, J. Cai, K. Le, F. Lianghuo, Y. Shimuzu, C.S. Lim, K. Subramaniam (eds), *The First Sourcebook on Asian Research in Mathematics Education: China, Korea, Singapore, Japan, Malaysia and India*. North Carolina: Information Age Publishing.

Banerji, R. and W. Wadhwa (2013). 'Every Child in School and Learning Well in India: Investigating the Implications of School Provision and Supplemental Help', in *India Infrastructure Report 2012*, IDFC Foundation. New Delhi: Routledge, pp. 52–63.

Batra, Poonam. (2006). 'Building on the National Curriculum Framework to Enable the Agency of Teacher's'. *Contemporary Education Dialogue*, 4(1).

———. (2009). 'Teacher Empowerment: The Education Entitlement – Social Transformation Traverse'. *Contemporary Education Dialogue*, 6 (2): 121–56.

———. (2013). 'Positioning Teachers in the Emerging Education Landscape of Contemporary India', in *India Infrastructure Report 2012,* IDFC Foundation. New Delhi: Routledge, pp. 219–31.

Bedamatta, R. (2014). 'Education in the Age of Reforms', *Economic and Political Weekly*, XLIX (51): 20, December 2014.

Bernstein, B. (1971). *Class, Codes and Control*: *Volume I, Theoretical Studies Towards a Sociology of Language.* London: Routledge, Kegan and Paul.

Bourdieu, P. and J.C. Passeron (1990). *Reproduction in Education, Society and Culture*. London: Sage Publications.

Bowles, S. and H. Gintis (1976). *Schooling in Capitalist America: Educational Reform and the Contradiction of Economic Life*. London: Routledge and Kegan Paul.

Chopra, R. and P. Jaffrey (eds.) (2005). *Education Regimes in Contemporary India*. New Delhi: Sage Publications.

Dowling, P. (1998). *The Sociology of Mathematics Education: Mathematical Myths / Pedagogical Texts*. London: The Falmer Press.

——— (2003). 'Discursive Saturation and School Mathematics Texts: A Strand from a Language of Description', in Paul Ernest (ed.), *Mathematics Education and Philosophy: An International Perspective.* London: The Falmer Press.

Duval, R. (1998). 'Geometry from a Cognitive Point a View', in C. Mammana and V. Villani (eds.), *Perspectives on the Teaching of Geometry for the 21st Century*. Dordrecht: Kluwer Academic Publishers, pp. 37–52.

———. (2000). 'Basic Issues for Research in Mathematics Education', in T. Nakahara and M. Koyama (eds.), Proceedings of the 24th Conference of PME, 1. Hiroshima: Nishiki Print Co. Ltd., pp. 55–69.

———. (2006). 'A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics', *Educational Studies in Mathematics,* 61: 103–31.

English, L. D. (2002). 'Priority Themes and Issues in International Research in Mathematics Education'. In L.D. English (ed.) *Handbook of International Research on Mathematics Education*. New Jersey: Laurence Erlbaum Associates.

Ernest, P. (1994). 'Introduction', in Paul Ernest (ed.), *Mathematics Education and Philosophy: An International Perspective.* London: The Falmer Press, pp. 1–8

———. (1998). 'The culture of the Mathematics classroom and the relations between the personal and public knowledge: An Epistemological Perspective', in F. Seeger, J. Voigt and U. Waschescio (eds), *The Culture of the Mathematics Classroom.* Cambridge: Cambridge University Press.

———. (2003). 'The Dialogical Nature of Mathematics', in Paul Ernest (ed.), *Mathematics Education and Philosophy: An International Perspective.* London: The Falmer Press.

Foucault, M. (1991). 'Governmentality', in G. Burchell, C. Gordon and P. Miller (eds.), *The Foucault Effect.* Brighton: Harvester Wheatsheaf.

Ghosh, J. (2011). 'Funding, the key', *Frontline*, 28(14): 10–11, July 2011.

Giroux, H. 1983. Theory and Resistance in Education, Westport, CT: Bergin and Garvey Press.

Heath, S.B. (1983). *Ways with Words: Language, Life and Work in Communities and Classrooms.* Cambridge: Cambridge University Press.

Jorgensen, R., P. Gates and V. Roper (2014). 'Structural Exclusion through School Mathematics: Using Bourdieu to understand Mathematics as a Social Practice', *Educational Studies in Mathematics* 87(2): 221–39. Doi: 10.1007/s10649-013-9468-4.

Jurdak, M. (2014). 'Socio-economic and Cultural Mediators of Mathematics Achievement and Between-school Equity in Mathematics Education at the Global Level', *ZDM—The International Journal on Mathematics Education*, 46(7). Doi: 10.1007/ s11858-014-0593-z.

Kaur, J. (2010). 'Developmental Changes in the Conceptual Understanding: A Study of 'Triangle' and 'Circle' in Classes V and VII'. Unpublished dissertation. New Delhi: Jamia Millia Islamia.

Khan, F. A. (2004). 'Living, Learning and Doing Mathematics: A Study of Working Class Children in Delhi', *Contemporary Education Dialogue,* 2: 199–227.

———. 2010. 'Social Dimensions of Mathematics Learning', in K. Subramaniam (ed.) *The EPIsteme Review: Research Trends in Science, Technology and Mathematics Education.* Delhi: Macmillan Publishers India.

———. (2012). 'The Social Implications of Mathematics Education: What Should be Taught and Why', in M. Kapur, H. Koot and M. E. Lamb (eds), *Developmental Psychology and Education: Bridging the Gap*. New Delhi: ICSSR, UK: ESRC and Netherlands: NOW.

Kumar, Krishna (2008). *A Pedagogue's Romance: Reflections on Schooling*. New Delhi: Oxford University Press.

Lerman, S. (2000). 'The Social Turn in Mathematics Education Research', in J. Boaler (ed.), *Multiple Perspectives on Mathematics Teaching and Learning*. Westport, CT: Ablex, pp. 19–44

Little, Angela W. (1988). *Learning from Developing Countries*, Monograph. University of London: The Institute of Education.

Majumdar, M. and Mooij, J. (2012). 'The Marks Race: India's Dominant Education Regime and New Segmentation', in C.E. Sleeter, S.B. Upadhyaya, A. Mishra, and S. Kumar (eds), *School Education, Pluralism and Marginality: Comparative Perspectives.* Delhi: Orient BlackSwan. pp. 207–46.

Manjrekar, N. (2003). 'Contemporary Challenges to Women's Education: Towards an Elusive Goal?', *Economic and Political Weekly*, 38 (43): 4577–82.

Minocha, D. (2013). *Understanding Teachers' Perceptions of Mathematics NCERT Textbooks of Grades 3, 4 & 5*. Unpublished field attachment report. Mumbai: Tata Institute of Social Sciences.

Nambissan, G. B. (2014). 'Sociology of School Education in India: A Review of Research 2000–2010', in Yogendra Singh (ed.) *Indian Sociology Volume 2, Development and Change*, ICSSR Research Surveys and Explorations. New Delhi: Oxford University Press, pp. 66–101.

Nambissan, G. B. and S. S. Rao (eds). (2013). *Sociology of Education in India: Changing Contours and Emerging Concerns*. New Delhi: Oxford University Press.

National Council for Educational Research and Training (2006). 'National Focus Group on Teaching of Mathematics', Position Paper. New Delhi: NCERT.

Payne, J.P. and J.B. Biddle (1999). 'Poor School Funding, Child Poverty and Mathematics Achievement', *Educational Researcher*. 28(6): 4–13, August–September 1999.

Piaget, J. (1958). *The Child's Construction of Reality*. London: Routledge and Kegan Paul.

———. (1965). *The Child's Conception of Number*. New York: W.W. Norton.

Pritchet, L. and A. Beatty (2012). 'The Negative Consequences of Overambitious Curricula in Developing Countries', Working Paper. Harvard: Kennedy School of Governance.

Rampal, A. and S. Mahajan (2003). 'A Study on School Effectiveness: UNICEF Education Programme Review (West Bengal)'. Paper presented at the National Review of the Quality Education Programme, Mimeographed. New Delhi.

Rana, Kumar (2006). 'Mid Day Meal and Primary Education: Prospectus and Challenges in West Bengal'. Paper presented at the seminar on Education and Inequality in Andhra Pradesh and West Bengal. Hyderabad: Centre for Studies in Social Sciences, 21-22 September.

Salam, J. (2014). 'Theft Corruption and Parental School Choice in Manipur', *Economic and Political Weekly*, XLIX(12).

Sarangapani, P. (2011). 'Teachers first', *Frontline*, 28(14): 13–15, July 2011.

Talib, M. (1992). 'Ideology, Curriculum and Class Construction: Observations from a School in a Working Class Settlement in Delhi', *Sociological Bulletin*, 41(1–2): 81–95.

Valero, P. and T. Meaney (2014). 'Trends in Researching the Socioeconomic Influences on Mathematical Achievement', *ZDM Mathematics Education*. Doi: 10.1007/s11858-014-0638-3.

Young, M. (2011). 'What are Schools for?', *Educacao, Sociedade & Culturas,* 32: 145–55.

Young, M. and Whitty, G. (eds.) (1977). *Society, State and Scholarship*. Lewes: Falmer Press.

Zevenbergen, R. (2000). 'Cracking the Code' of Mathematics Classrooms: School Success as a Function of Linguistic, Social and Cultural Background', in J. Boaler (ed.), *Multiple Perspectives on Mathematics Teaching and Learning*. Westport: Ablex, pp. 201–24.

Acknowledgement

Grateful thanks to Geetha Nambissan, Indra Sengupta and to the Transnational Research Group for giving me the opportunity to adapt this work to a larger audience. Thanks also to Jasneet Kaur for her very helpful inputs.